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**MODELING DYNAMICALLY COUPLED FLUID-DUCT  
SYSTEMS WITH FINITE LINE ELEMENTS**

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Structural analysis of piping systems, especially dynamic analysis, typically considers the duct structure and the contained fluid column separately. Coupling of these two systems, however, forms a new dynamic system with characteristics not necessarily described by the superposition of the two component system's characteristics. Methods for modeling the two coupled components simultaneously using finite line elements are presented. Techniques for general duct inter-sections, area of direction changes, long radius bends, hydraulic losses, and hydraulic impedances are discussed. An example problem and results involving time transients are presented. Additionally, a program to enhance post-processing of line element models is discussed.

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# ***Modeling Dynamically Coupled Fluid-Duct Systems with Finite Line Elements***

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## **Abstract**

Structural analysis of piping systems, especially dynamic analysis, typically considers the duct structure and the contained fluid column separately. Coupling of these two systems, however, forms a new dynamic system with characteristics not necessarily described by the superposition of the two component system's characteristics. Methods for modeling the two coupled components simultaneously using finite line elements are presented. Techniques for general duct intersections, area or direction changes, long radius bends, hydraulic losses, and hydraulic impedances are discussed. An example problem and results involving time transients are presented. Additionally, a program to enhance post-processing of line element models is discussed.

## **Introduction**

Structural analyses of piping systems are usually accomplished with finite line element models to which estimated fluid loads are applied. Under transient conditions, however, dynamic coupling often occurs between the pipe and the contained fluid such that the two can no longer be considered separately. In the analyses of Space Shuttle Main Engine propellant feedlines and ducts, the need for modeling structural dynamic response of coupled fluid-duct systems has led to new analysis methods. These methods allow simultaneous analysis of the structure and contained fluid column by representing both as overlaying strings of line elements. This requires the fluid to be treated as one dimensional; an assumption considered adequate from the structural analyst's perspective. The same basic approach has been previously explored (Reference 1), but the specifics detailed herein were developed independently and contain some unique features which represent an expanded application of the underlying principles. The bulk of the presented methods deals with maintaining the correct force transfer between the duct and fluid elements. The theory includes treatment of forces transferred at general intersections or direction/area changes, forces transferred at long radius bends, forces due to losses, and terminal hydraulic impedances. Post-processing animations are also discussed.

## **Dynamic Approach**

In the presence of both steady-state and transient dynamics, it is desirable to consider the two separately and superimpose their results for a complete answer. In the proposed method, the steady-state dynamics are analyzed with a quasi-static

approach for which the FEA representation of the fluid column assumes static equilibrium, but is understood to represent some steady-state dynamic condition. Forces transferred by the quasi-static model accurately mirror those of the true steady-state dynamic condition except that head losses are usually ignored. If a loss is considered significant, then the steady-state analysis should include forces applied by the analyst to the fluid column and/or structure as a correction. Losses will be addressed later in greater detail.

The underlying assumption of the transient analysis is that transient dynamic loads produce the same response whether applied to a quasi-static system or to a genuinely steady-state dynamic system. Under transient loads, velocities and loads experienced by the FEA representation of the fluid are understood to add vectorially to the corresponding steady-state values. Forces due to losses are generally a function of velocity squared, and therefore should have a transient component as well. Discussion of a method for estimating transient losses will also be deferred, but will be built on the assumption that transient velocities are small compared to steady-state velocity.

By analyzing steady-state and transients separately, a quantifiable error is introduced in the rate at which pressure perturbances are propagated up and downstream. In reality, pressure wave propagation should be a superposition of acoustic and steady-state velocities. Error due to the absence of a steady-state velocity is dismissed, however, since acoustic velocities dwarf flow velocities within the scope considered here.

## Modeling a Fluid Column and Coupled Straight Pipe

Analysis of a duct or pipe-line structure with finite line elements is commonplace. The fluid coupling methods presented herein are intended as an addition to the standard structural model of a duct. Therefore, definition of the structural portion of the coupled system requires no particular deviation from standard practice, nor will it be discussed in any detail.

Treatment of the contained fluid column is one dimensional and therefore representable by structural rod elements (structural members that carry only tension/compression). Properties of the rod elements are based on properties of the fluid column they represent. Mass density of the rod should equal density of the fluid, and cross-sectional area of the rod should equal the cross-sectional area of fluid column. Young's Modulus of the rod corresponds to Effective Bulk Modulus,  $Be$ , of the fluid, which is calculated from Bulk Modulus,  $B$ , corrected for radial stiffness of the pipe as shown below.

$$Be = (BEt)/(2BR + Et) \quad (1)$$

where  $E$  = Young's Modulus of pipe  
 $t$  = pipe wall thickness  
 $R$  = nominal radius of pipe

For duct cross-sections more complex than cylindrical pipe, such as bellows,  $Be$  may be calculated by methods such as demonstrated in Reference 2. With these fluid/structural analogies in place, the axial stress in the rod is taken to equal the total pressure, static plus dynamic, in the fluid column.

The validity of this approach can be confirmed by a simple model of a fluid column constrained in all but the axial direction along its length and completely constrained at one end. FEA modal analysis should match hand calculated open-closed organ pipe frequencies.



For the sake of conveniently coupling fluid to pipe, it is beneficial to adopt certain conventions in defining the two coincident strings of line elements which represent the duct structure and the fluid column. First, it is convenient that nodes on one element string have its nodes coincident with nodes of the other string, such that pairs of corresponding fluid/structural nodes exist. Each node pair should be coupled such that they move independently in the direction corresponding to flow, but are otherwise rigidly joined. This is illustrated schematically in Figure 1, and may be accomplished with either Multi-Point-Constraint (MPC) equations or zero length springs. As a convenience to defining the coupling, nodal degrees of freedom, especially those of the fluid node, should be aligned with one axis in the direction of flow. As a matter of convention, it will be assumed herein that the nodal Z axis is so oriented.

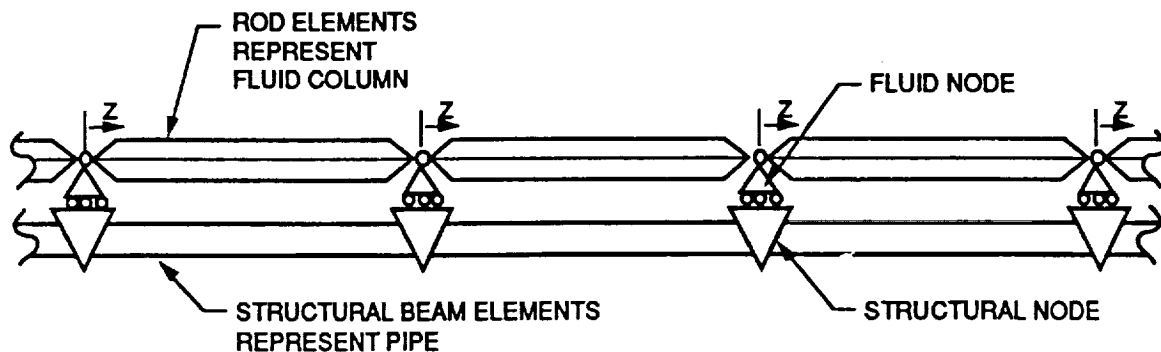


Figure 1. FEA of Straight Duct and Coupled Fluid Column

### Coupling at Direction/Area Changes

At any point where the duct contains a bend, an intersection, or a change in flow area, considerations beyond those described thus far must be applied in order to correctly account for the transfer of forces between the fluid column and the duct structure at that location. For now, the discussion will be limited by the assumption that the duct feature in question can be represented as a point along the duct path. If the size of the feature is considerable compared with the overall model, namely, the case of a very long radius elbow, an alternate approach described later may be more appropriate.

Figure 2 illustrates a pipe intersection involving all the features under consideration. (Planar geometry is not necessary, but is used here for clarity.) Mass continuity between the three fluid columns and force transfer between the fluid and the duct can be imposed with a single MPC equation involving the Z translations of each fluid node and the translational degrees of freedom of the intersection's structural node. The equation is derived by treating the intersection's structural node as if it were another incoming fluid branch oriented and sized such that all the Pressure-Area forces acting on the intersection added vectorially to zero.

A generalized approach, as applied to Figure 2, proceeds as follows. Each of the three branches entering the intersection should be modeled as described previously, and each branch should have its own fluid node at the point of intersection. For convenience, direct the Z axes of the intersection's three fluid nodes *into* the intersection. Let the vectors  $\mathbf{A}_i$  equal the flow area of branch  $i$  times the unit Z vector of branch  $i$ 's fluid node at the intersection. Each  $\mathbf{A}_i$  should be expressed in the reference frame defining the nodal displacements of the

intersection's structural node. Now define a vector  $A_S$  according to

$$A_S + \sum_{i=1}^3 (A_i) = 0 \quad (2)$$

Having defined  $A_S$ , the MPC equation enforcing mass continuity and force balance is

$$(A_{sx} \cdot X_s) + (A_{sy} \cdot Y_s) + (A_{sz} \cdot Z_s) + \sum_{i=1}^3 (A_i \cdot Z_i) = 0 \quad (3)$$

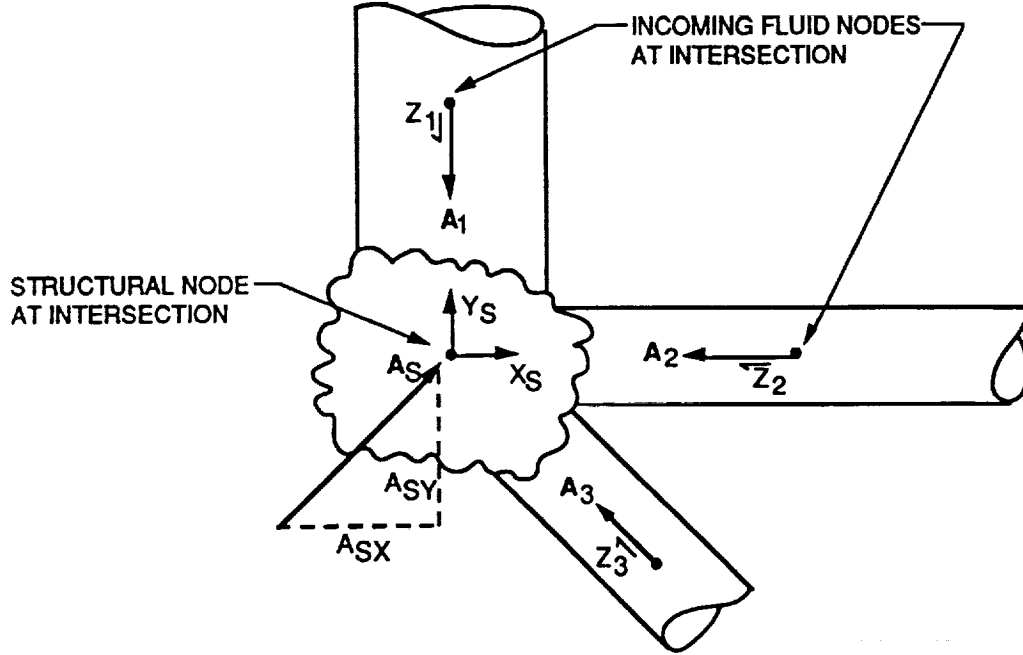


Figure 2. Duct Intersection and Area Change Example

Of course, equations 2 and 3 are applicable to any number of incoming branches. If applied to one branch, a capped end is defined. If applied to two branches, an elbow is defined.

For some analysis software, it may be necessary to enforce these constraints with large stiffness values rather than an MPC equation. In effect, the stiffness equations form the intersection element defined below for  $n$  branches intersecting at a point. Here,  $K_1$  is an arbitrary multiplier just large enough to make fluid at the point of intersection relatively incompressible.

$$\begin{pmatrix} F_{z1} \\ F_{z2} \\ F_{z3} \\ F_{xs} \\ F_{ys} \end{pmatrix} = K_1 \begin{bmatrix} A_1 \cdot A_1 & A_1 \cdot A_2 & A_1 \cdot A_3 & A_1 \cdot A_{sx} & A_1 \cdot A_{sy} \\ A_2 \cdot A_1 & A_2 \cdot A_2 & A_2 \cdot A_3 & A_2 \cdot A_{sx} & A_2 \cdot A_{sy} \\ A_3 \cdot A_1 & A_3 \cdot A_2 & A_3 \cdot A_3 & A_3 \cdot A_{sx} & A_3 \cdot A_{sy} \\ \text{symm.} & & & A_{sx} \cdot A_{sx} & A_{sx} \cdot A_{sy} \\ & & & A_{sy} \cdot A_{sx} & A_{sy} \cdot A_{sy} \end{bmatrix} \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ X_s \\ Y_s \end{pmatrix} \quad (4)$$

## Long Radius Elbows

In some cases, the radius of a bend is not negligible compared to the total length of duct being modeled. Figure 3 shows such a bend, encompassing an angle  $\theta$ , as modeled with  $n$  fluid/structural node pairs spaced evenly between the end points of the bend. As with straight pipe, the fluid and structural nodes should move independently in the direction tangent to the direction of flow, but be rigidly coupled perpendicular to flow as illustrated. This arrangement accurately models the forces transferred, however, the stresses in the rod elements will exceed the total pressure in the fluid. This can be corrected by adjusting the properties of the rods so as to maintain correct axial stiffness and achieve correct stresses. The basic properties of the rods in the bend, e.g.  $A$ ,  $E$ , and  $\rho$ , should be adjusted to  $A'$ ,  $E'$ , and  $\rho'$  according to

$$A' = A / \sin(\theta / (n+1)) \quad (5)$$

$$E' = E \cdot \sin(\theta / (n+1)) \quad (6)$$

$$\rho' = \rho \cdot \sin(\theta / (n+1)) \quad (7)$$

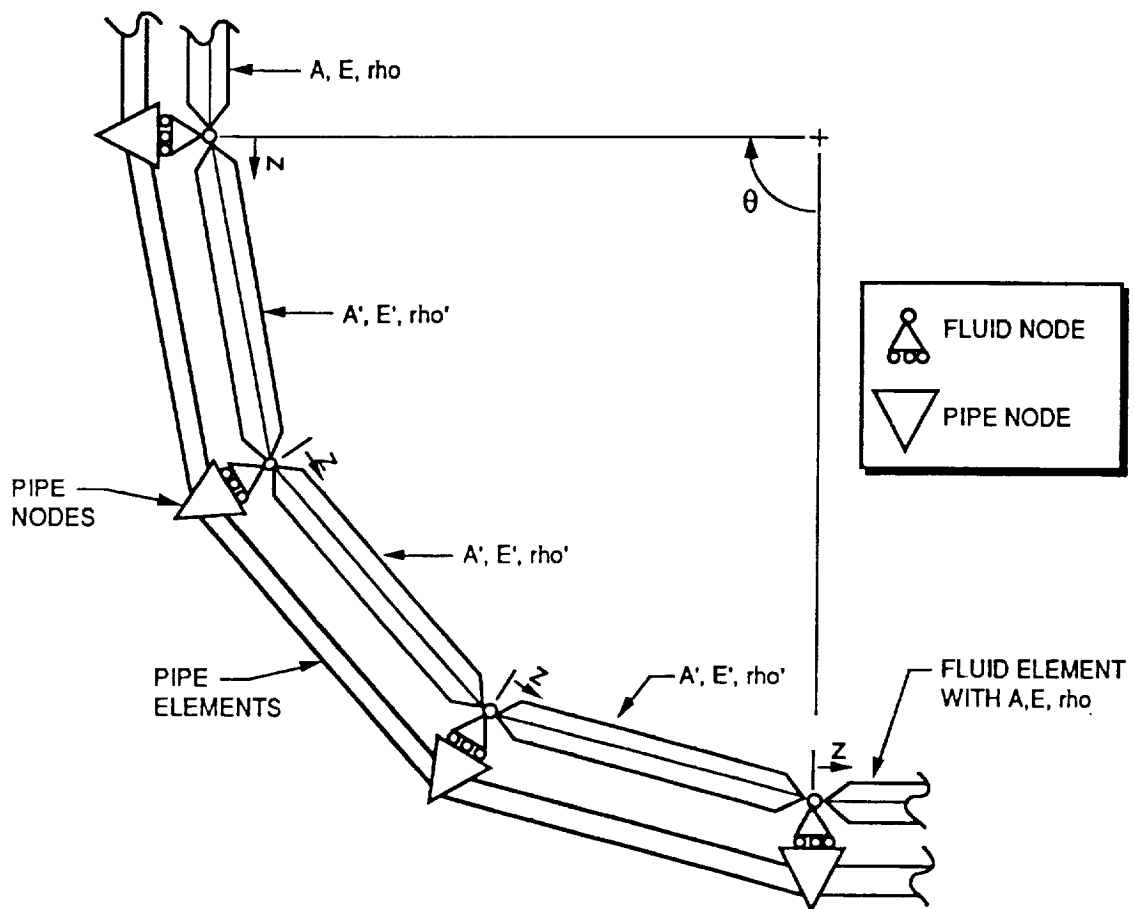


Figure 3. FEA of Curved Duct and Coupled Fluid Column

## Losses

In addition to force transfer between fluid and duct at direction/area changes, head losses may play a significant role in how forces are distributed throughout the system. As previously stated, steady-state losses must be accounted for by the analyst. This is accomplished with equal and opposite forces applied to the fluid column in the upstream direction, and to the structure in the downstream direction. The magnitude of the loss, where steady-state velocity is a given quantity, must be determined as shown below, where  $C$  is obtained from reference data.

$$F_{Loss} = A \cdot C \cdot \rho \cdot (V_{ss})^2 \quad (8)$$

Figure 4 illustrates an elbow under steady-state conditions with an assumed loss. The mass and force balance equations would make pressure in the fluid column the same both up- and downstream of the elbow. The additionally applied forces for loss allow the fluid column downstream to see a reduced pressure while maintaining mass continuity.

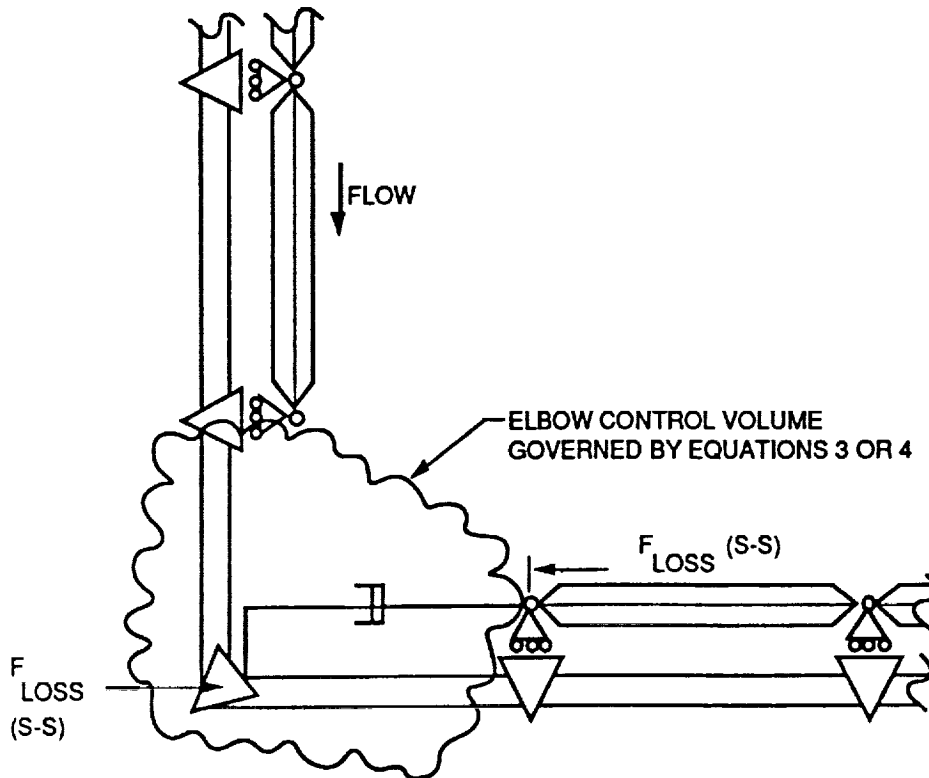


Figure 4. FEA of Elbow with Losses Considered

In the presence of transients, the total velocity,  $V_{ss} + V_t$ , should produce a Force of loss equal to the sum of the steady-state loss,  $F_{ss}$ , and a transient component of loss,  $F_t$ , as shown below.

$$F_{Loss} = F_{ss} + F_t = A \cdot C \cdot \rho \cdot (V_{ss} + V_t)^2 \quad (9)$$

Assuming that the loss variable  $C$  does not appreciably change between velocities  $V_{ss}$  and  $V_{ss} + V_t$ , substituting equation 8 into 9 and solving for  $F_t$  gives

$$F_t = A \cdot C \cdot \rho \cdot (2 \cdot V_{ss} \cdot V_t + V_t^2) \quad (10)$$

If a further assumption is made that  $V_t^2$  is insignificant compared with  $2 \cdot V_{ss} \cdot V_t$ , the transient force of loss can be estimated as

$$F_t = (2A \cdot C \cdot p \cdot V_{ss}) \cdot V_t \quad (11)$$

which is a linear damping term. As long as  $C$  is constant, the above estimation is 90% accurate for  $V_t < (.2 \cdot V_{ss})$ . The damper implied above should be affixed at the point of loss between the structural node and the downstream fluid node as seen in Figure 4. The damper increases the force of loss for a transient velocity that increases flow velocity magnitude and decreases the force of loss for a transient velocity that decreases flow velocity magnitude. Unfortunately, the above treatment does not account for the fact that much of the loss represented by published values of  $C$  accounts for irreversible losses several diameters downstream of the elbow, thus reducing the static pressure in the fluid without causing an equal and opposite load on the duct. None-the-less, the above method does provide an estimate of force distribution due to transient losses.

### Terminal Hydraulic Impedances

The ducts addressed to date have all terminated at the inlet to a rocket engine or an accumulator device which provided a quantifiable impedance to flow. The Hydraulic impedance is expressed as compliance, resistance, and inertance, for which the mechanical equivalents are stiffness, resistance, and mass, respectively. For the steady state analysis, the terminal end of the fluid column can be fixed, thereby precluding the need to account for the impedance. In the transient analysis, however, the fluid column must terminate at a spring/mass/damper system, as shown in Figure 5, with  $K$ ,  $M$ , and  $D$  defined in terms of compliance, mass, and resistance as follows:

$$K = \Gamma \cdot A / \text{Compliance} \quad (12)$$

$$M = \Gamma \cdot A^2 \cdot \text{Inertance} \quad (13)$$

$$D = \Gamma \cdot A^2 \cdot \text{Resistance} \quad (14)$$

Where  $\Gamma$  is weight of density of fluid

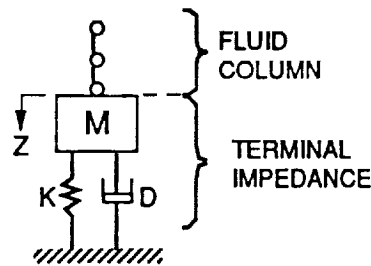


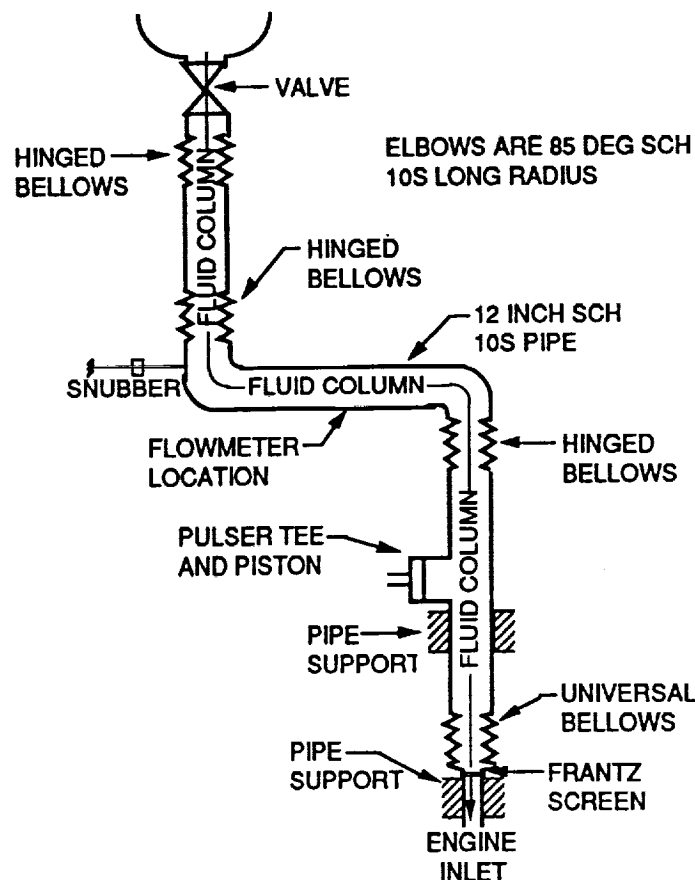
Figure 5. Mass-Spring-Damper System to Model Terminal Hydraulic Impedance

Note that these impedance terms are often frequency dependent. This requires the analyst to choose coefficients carefully, possibly analyzing frequency domain transients piece-wise over the frequency range of interest.

### Example Problem

As a demonstration of the modeling concept, a transient analysis is presented based on work performed for NASA's Marshall Space Flight Center.

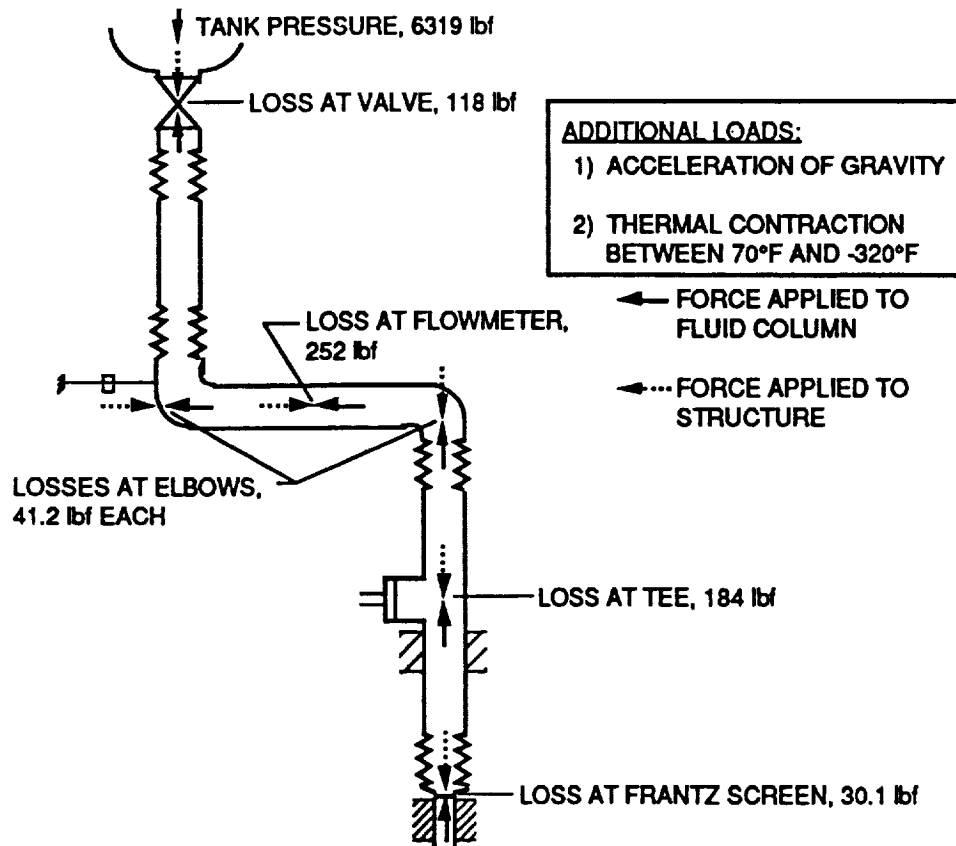
The Center is home to Technology Test Bed, a facility for test firing Space Shuttle Main Engines. The problem addresses the possibility of incorporating an oscillating piston in a side branch of the main Liquid Oxygen feedline in order to agitate pressure at the engine inlet (See Figure 6.) This test procedure, known as pogo pulsing, simulates the effect of space craft vibrations on the propellant systems, which, in turn, has been observed to cause and couple with engine thrust oscillations. The immediate problem was to predict pogo pulsing's effect on the facility feedline.



*Figures 6. Schematic of TTB Liquid Oxygen Feedline*

The coupled fluid-duct model of the feedline incorporates many of the modeling features described herein. At both elbows and the pulser tee, stiffness matrices representing general area/direction changes were applied. Losses at valves, flowmeters, etc. were modeled with linear dampers, the damping values based partially on pressure data recorded during operation. The terminal hydraulic compliance of the engine was modeled with a spring, but, for the sake of this example, its frequency dependence and any accompanying hydraulic resistance were ignored.

Quasi-static analysis of steady-state operation was relatively straightforward. The applied loads, including those to account for losses, are shown in Figure 7.



**Figure 7. Loss Loads Applied to Feedline Model for Quasi-static Analysis of Steady-state Conditions**

The results of three Modal analyses, shown in Table 1, demonstrate the inadequacy of analyzing the fluid and structural system separately. The first set of modes considers the fluid acoustics only, as if the duct were perfectly rigid (except for radial expansion accounted for by equation 1). The second set of modes considers duct flexibility, but treats the fluid only as added mass. The last set of modes, which accounts for the fluid-duct coupling, is clearly more than the simple superposition of the first two cases.

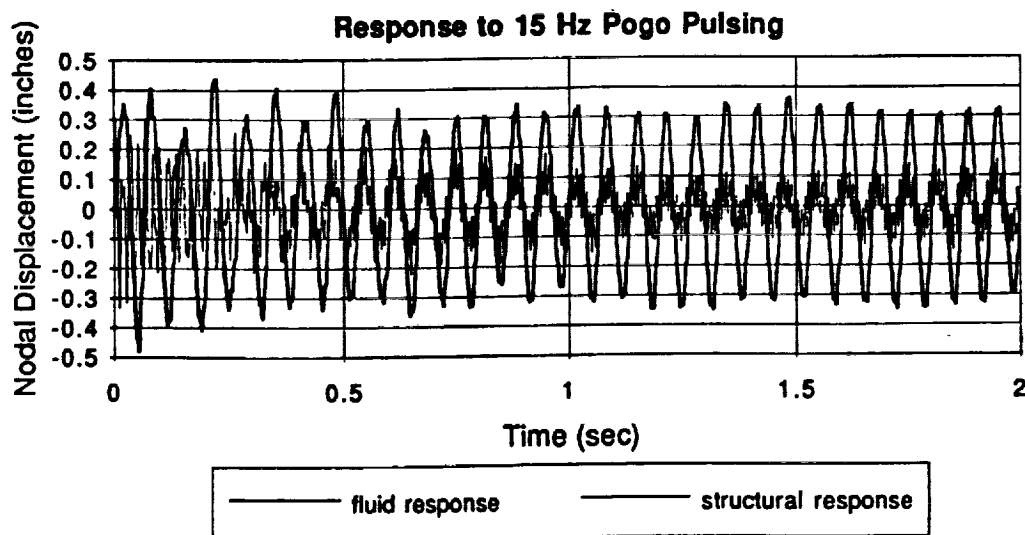
Modes from the coupled system were used in two transient analyses simulating the start-up of pogo pulser operation at 15 Hz and 35 Hz. Both analyses were driven by enforcing sinusoidal displacement of the piston node. Figures 8 and 9 show response of the system presented as nodal displacement of a point in the fluid column near the engine inlet, and displacement of a node on the duct near the downstream elbow. The transient response to 15 Hz Pogo pulsing quickly settles to a steady-state containing higher frequency components - as might be expected. As might also be expected, response to 35 Hz pogo pulsing exhibits resonance with the coupled system mode at 34.9 Hz, amplifying the input signal.

Obviously, any analysis of Pogo pulsing based on the separate models of the fluid and duct would have been much more crude. The fact that 35 Hz was a critical system frequency would have remained unknown since it shows up in neither of the first two columns of Table 1. Even at non-critical frequencies, the model can be used to quantify structural stresses as a function of the oscillating pressure seen at the engine inlet and/or the stroke of the piston. Hopefully, this

method will prove applicable and useful in the structural analysis of other coupled fluid-duct systems to address problems involving waterhammer, hydraulic control, accumulator systems, etc. Already, the method has been employed in the analysis of other NASA related systems such as the duct and accumulator, components of the space shuttle main engine, seen in Figures 12 and 13.

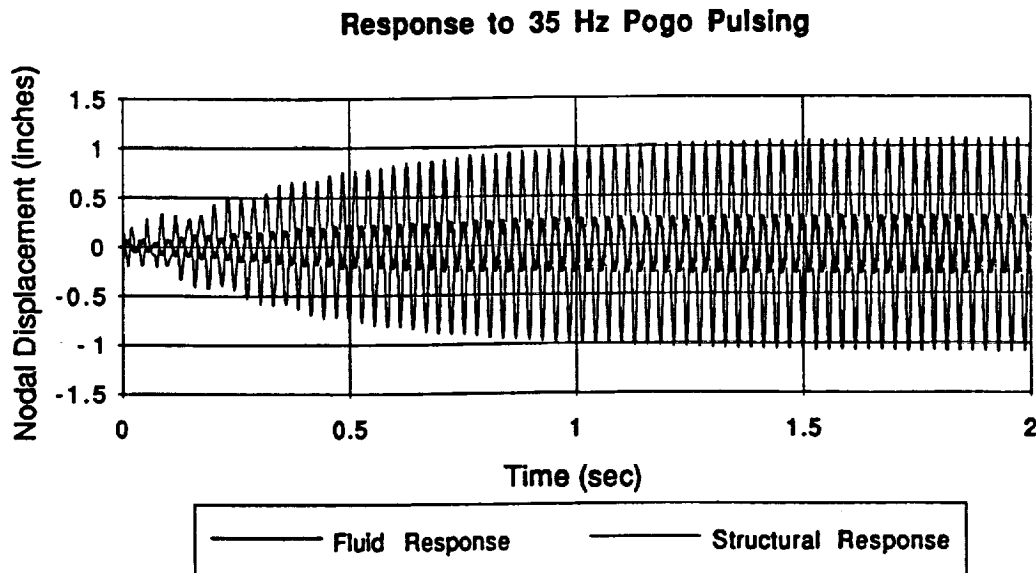
*Table 1. Comparison of Modal Results*

Fluid Acoustic Modes, Rigid Duct (Hz)	Duct Structural Modes, "Frozen" Fluid (Hz)	Coupled Fluid-Duct System Modes (Hz)
1.8		1.8
	6.9	6.1
29.6		14.3
	32.4	22.1
59.1		26.6
	66.1	32.2
	77.4	34.9
88.5		62.5
	92.6	71.8
	97.6	84.8
118.0		89.8
	124.2	93.6
	133.6	98.3
	142.2	118.6
147.6		130.9
177.0		143.8
	181.1	145.7
	192.9	162.7
205.2		179.4
		191.5



*Figure 8. Transient Feedline Response to Pogo Pulsing Start-up at 15 Hz*

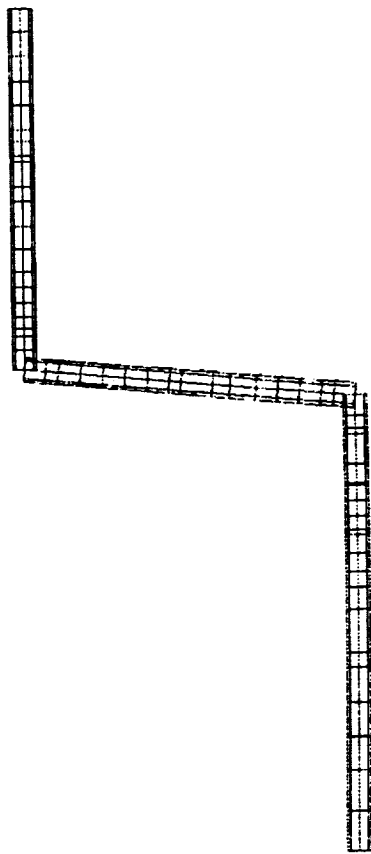




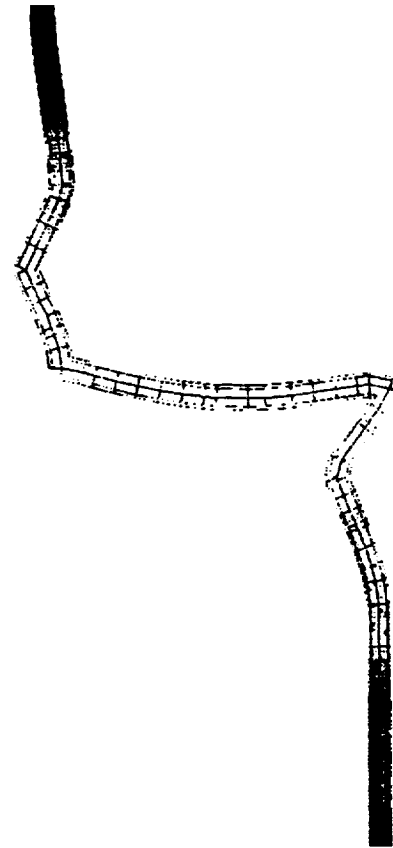
*Figure 9. Transient Feedline Response to Pogo Pulsing Start up at 35 Hz*

### Post-Processing Animations

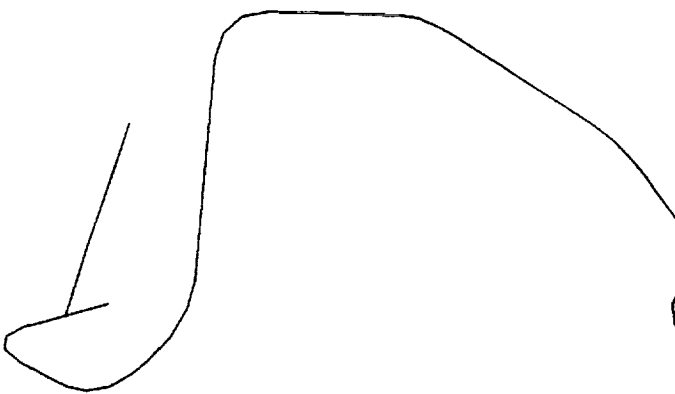
As is common to all line element models, deformed geometry animations of models such as described herein are bland. The information sought, namely pressure distribution in the fluid and its correlation with structural response, is difficult to convey visually with line element plots. In order to enhance understanding of the solution, a Fortran program called DUCT6D was written which, in conjunction with PATRAN 2.5, simultaneously presents the model, its structural deformations, and fluid pressures in a color animation. DUCT6D converts the line elements into rings of shells elements so that the circular cross section of the duct can be visually perceived. The color of the shells changes with the pressure in the fluid column. Deformation of the shell model mirrors the extrapolated displacements and rotations of the line model. DUCT6D will produce both modal and transient animations. DUCT6D is available from COSMIC (See NASA Tech Briefs, August 1993, p. 33) in a format compatible with output from the EAL analysis code, however, minor editing of input formats should adapt it to other specific analysis packages. Figure 10 shows the undeformed Lox feedline from the example problem as depicted with DUCT6D. Figure 11 shows the 35 Hz mode shape from the example problem as depicted with the benefit of DUCT6D, where the darker color indicates higher pressure. Figures 12 and 13 further demonstrate DUCT6D's usefulness in visualizing line element models. Figure 12 is a plot of a coupled fluid-duct model of a duct/accumulator system found on the space shuttle main engine. Figure 13 is the same model as enhanced with DUCT6D.



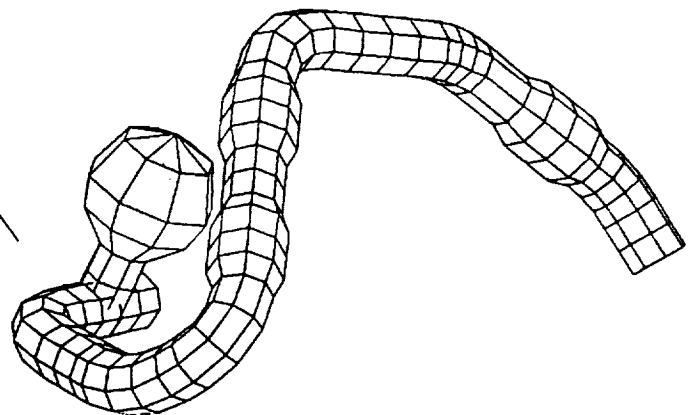
*Figures 10. Feedline Model as  
Depicted with DUCT6D*



*Figures 11. Mode 7 of Feedline Model,  
34.9 Hz, as Depicted with DUCT6D*



*Figures 12. Line Element SSME  
Lox Duct*



*Figures 13. SSME Lox Duct  
After DUCT6D*

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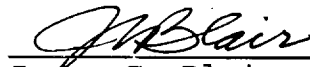


APPROVAL

MODELING DYNAMICALLY COUPLED  
FLUID-DUCT SYSTEMS WITH FINITE  
LINE ELEMENTS

BY J.B. SAXON

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.



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